**Project Report**

**Online Payment Fraud Detection Model**

In Partial Fulfillment

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**Abstract**

Online payment fraud happens when someone illegally obtains another person's payment information and uses it to make unauthorized transactions or purchases. This project report presents a step-by-step data science approach for detecting fraudulent online payment transactions. The project utilizes a synthetic dataset [12][14], which simulates a series of payment transactions. The report provides a detailed account of the dataset, followed by in-depth explanations of exploratory data analysis, data cleaning, transformation, feature generation, feature selection, and model development. It also features a comparative analysis of four anomaly detection algorithms, evaluating their performance metrics after being trained on the processed dataset. The report concludes by outlining a dynamic solution for detecting online payment fraud.

***Keywords****:* Online Frauds, Fraud Detection, Anomaly Detection, Data Wrangling, Feature Generation

**1. Introduction**

**1.1 Motivation**

As online transactions become more popular across platforms like e-commerce, mobile payments, and online banking, the financial industry is growing increasingly concerned about the prevalence and prevention of online payment fraud. Payment fraud generally falls into three categories: conventional fraud (such as stolen, fake, or counterfeit cards), online fraud (such as fraudulent merchant sites), and merchant-related fraud (including merchant collusion and triangulation schemes) [9]. The US is the country most susceptible to fraud, with 34% of consumers reporting they were likely victims of fraud—a percentage that is likely even higher today [5]. According to an estimate by Juniper Research, online payment theft could result in a staggering loss of over $200 billion between 2020 and 2024, underscoring the critical need for effective fraud detection systems [8]. Moreover, the research revealed that to effectively combat increasing fraud, fraud prevention vendors need to strategically implement the right combination of verification tools at the most critical points in the customer journey. However, achieving this will require substantial capabilities [8].

**Conclusive statement:** The aforementioned statistics highlight the need for a dynamic approach to fraud detection that combines data science concepts with machine learning techniques to effectively address the frequent occurrence of online fraud.

**1.2 Literature Survey**

The available literature related to detection of online frauds using artificial intelligence points to a fact that the researchers classify this challenge in the domain of anomaly detection. [1] uses the clustering method to categorize cardholders into high, medium, and low transaction amount groups through range partitioning, and then apply the Sliding-Window method to aggregate transactions into these groups and extract features from the window to identify cardholders' behavioral patterns. [4] utilizes a random forest approach by combining around 100 decision tree classifiers for detecting anomalous behaviour. [6] compares various machine learning algorithms, including Logistic Regression, Decision Trees, and Random Forest, to determine which one performs best and could be adopted by credit card merchants for identifying fraudulent transactions. [7] utilizes a data mining and machine learning tool named WEKA in tandem with data preprocessing techniques. [10] compares the performance of various algorithms such as Random Forest, Naïve Bayesian, Logistic Regression, SVM, kNN, Decision Trees and GBM. All of these papers [1][4][6][7][10] utilize a time series-based transactions’ dataset for their analytics and model. Distinct data mining techniques were applied in these research papers. [6] executes random sampling and feature selection over the dataset before proceeding to modelling phase. [1] aggregates the transactions into groups based on their transaction’s amounts. [7] uses the concept of 10-fold cross validation as validation methodology.

**2. A closer look: Online Payment Frauds Dataset**

**2.1 Data Excavation**

The dataset utilized for generating a balanced solution to the challenge of fraud detection was borrowed from [12]. The account holder of the dataset uploaded on [12] had an aim of identifying online payment fraud using machine learning techniques. However, one of the discussions [13] posted on [12]’s Kaggle account revealed that the dataset was originally created by [14]. [14] is the original fetcher of the utilized dataset in this project. [14] introduces a synthetic dataset created with the PaySim simulator as a solution to this problem. PaySim utilizes aggregated data from a private dataset to generate a synthetic dataset that mirrors typical transaction activity while incorporating malicious behaviors [14]. Therefore, the entire understanding of the dataset following this statement has been executed based on the information provided by [14].

**2.2 Schema & First Look Characteristics**

The utilized dataset had following characteristics as per [14] (the original data fetcher):

|  |  |
| --- | --- |
| **Data Parameter** | **Value** |
| Byte size (while downloading) | 186 MB |
| Number of rows (First look) | 6,362,620 (Around 6.36 million records) |
| Number of columns (First look) | 11 |
| Target Column (As mentioned by [12] and [14] ) | isFraud |
| Type of data | Synthetic (generated artificially) |

The schema of the dataset along with the description of columns as per [14] is as follows:

|  |  |  |
| --- | --- | --- |
| **Column Name** | **DataType** | **Description** |
| step | Int64 | Maps a unit of time in the real world. In this case 1 step is 1 hour of time. Total steps 744 (30 days simulation). |
| type | String | Type of online transaction: CASH-IN, CASH-OUT, DEBIT, PAYMENT and TRANSFER |
| amount | Float64 | The amount of the transaction |
| nameOrig | String | Customer starting the transaction |
| oldbalanceOrg | Float64 | Balance before the transaction |
| newbalanceOrig | Float64 | Balance after the transaction |
| nameDest | String | Recipient of the transaction |
| oldbalanceDest | Float64 | Initial balance of recipient before the transaction |
| newbalanceDest | Float64 | The new balance of recipient after the transaction |
| isFraud | Int64 | Target attribute: This is the transactions made by the fraudulent agents inside the simulation. |
| isFlaggedFraud | Int64 | The business model aims to control massive transfers from one account to another and flags illegal attempts. |

**3. Proposed Solution**

**3.1 Executed Ideation**

Various insights were fished out via the aforementioned research papers and links. The primary focus of every research paper was to go for a pure machine learning based approach. The focus on data pre-processing was kept minimal and major limelight of research was thrown on prediction algorithms. So, there was an idea – What if we apply the “Pareto Principle” [11] for solving this challenge? We can transfer our 80% focus on data wrangling, followed by 20% focus on results and machine learning algorithms. Fraud analytics and detection can be executed in a better way if the training data is made simple and easy to understand for the algorithms. The idea of running this project stems from the data exploration step which would involve analyzing the data w.r.t to the target attribute. This dataset is an example of anomaly detection datasets which generally have low target rates. The first look target rate of this dataset is 0.19%. Thus, it was decided to execute exploratory data analysis followed by a data cleaning process which would involve removal of unnecessary rows. This step will be followed by a feature generation process which would involve generating additional processed data columns from the existing ones which will provide a better fit to the models. The next step will involve splitting the dataset in two parts: [1] Training dataset and [2] Validation dataset. The training set will be down sampled to up the target rate to 5%. This down sampled training dataset will be exposed to 3 feature selection checks: a) Low Variance Check b) Covariance Check and c) Boruta Feature Selection Algorithm [15]. These checks are capable of filtering off the features which don’t provide a good fit for model development. Having passed the aforementioned checks, the training dataset with processed features will be normalized and utilized for training 4 anomaly detection algorithms: a) Isolation Forest [16] b) One Class SVM [17] c) Autoencoder Neural Network [18] and d) Elliptical

Envelope [19]. This process has ensured the 80% focus of the process in data mining and exploratory analysis followed by 20% focus in the domain of anomaly detection algorithms.

**3.2 Implementation Workflow**

**A diagram of a flowchart

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**4. Exploratory Data Analysis**

The first step in our journey involved analyzing the data to its core with respect to the target attribute. The utilized dataset had “isFraud” column which stores a series of target variables. However, [14] mentions that the column “isFlaggedFraud” needs to be focused upon first as it contains the transactions which were cancelled. The documentation says that these transactions should not be considered for further analytics. Removal of these 16 rows also removes the 16 positive target samples from the dataset. The next step was analyzing the “isFraud” column with respect to various parameters. The results of executing EDA over “isFraud” column are as follows:

|  |  |  |
| --- | --- | --- |
| **Comparison Parameter** | **isFraud=0** | **isFraud=1** |
| Total number of rows (count of transactions) | 6,354,407 (Around 6.3 M) | 8197 |
| Unique values in TYPE column | PAYMENT, DEBIT, CASH\_OUT, TRANSFER, CASH\_IN | TRANSFER, CASH\_OUT |
| Average value of executed transactions amount | 178, 197.04172740763 (Around 180 K) | 1,461,343.157758936 |
| Coefficient of variance of transaction amounts | 3.345942085050187 | 1.6403036828832473 |
| Average value of oldbalanceOrg column | 832828.7117272632 | 1637627.6859241184 |
| Coefficient of variance of oldbalanceOrg column | 3.4666720655500334 | 2.1543965997717454 |
| Average value of newbalanceOrg column | 855970.2281088118 | 177508.20801268757 |
| Coefficient of variance of newbalanceOrg column | 3.4171596962105553 | 10.790362135648685 |
| Average value of oldbalanceDest column | 1101420.8745693793 | 545311.9582115408 |
| Coefficient of variance of oldbalanceDest column | 3.4171596962105553 | 6.124181220726873 |
| Average value of newbalanceDest column | 1224925.6845631592 | 1282205.5214859096 |
| Coefficient of variance of newbalanceDest column | 2.9992151819666044 | 3.051165030899726 |
| Number of Merchants | 2151495 | 0 |

|  |  |
| --- | --- |
| **A blue and orange pie chart  Description automatically generated** | **a)** Merchants were only found at destination (amount receiver’s end)  **b)**  Majority of transactions – around 66% involved customers  **c)** Every transaction involving merchants was found to be legitimate. |
| **A graph of blue and orange bars  Description automatically generated** | **a)** The average transaction amount in case of fraudulent cases was 8 times as compared to fraudulent cases.  **b)** The old balance at origin was more in fraudulent cases. This means that fraud cases are likely to occur more with accounts having higher balances.  **c)** The balance at origin after the transaction was way less in case of frauds. This means fraudulent transactions suck up majority of balances at origin. |
| **A graph of different colored bars  Description automatically generated** | **a)** The coefficient of variance of new balance at origin is way more in fraudulent cases. This means the fraudulent transactions cause the source balance to drain to a great extent.  **b)** The coefficient of variance at destination end was found more in cases of fraudulent cases. This means the fraudulent transactions increase the balance at destination to a higher extent. |

**5. Data Wrangling**

**5.1 Data Cleaning**

The insights obtained from executed EDA steps were aggregated and a forward strategy of processing the data via cleaning and transformation was decided. The first step involved cleaning the data by removing the irrelevant rows with respect to the current dataset and its documentation [12][14]. The first step of data cleaning was recommended in [14] which involved removal of all transaction rows having “isFlaggedFraud” as 1. There were 16 rows such rows which were removed as the first step to data cleaning. The next step involved removing the transaction rows which had merchants at destination end. It was found via exploratory data analysis that none of the merchants had anything to do with fraudulent transaction. Thus, we removed all transaction rows involving merchants which reduced the number of non-anomalous targets (isFraud=0). Then, the string columns of the dataset except “type” were removed which were : 'step', 'nameOrig', 'destCustomerType', 'nameDest' and 'isFlaggedFraud'. This reduced the dataset to numerical columns and the target attribute (isFraud).

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**5.2 Data Encoding & Transformation**

The cleaned dataset still had a string column – “type”. The “type” column was subjected to a process of label encoding where the unique values - DEBIT, CASH\_OUT, TRANSFER, CASH\_IN were replaced by numbers 6, 4, 2 and 8 respectively. The dataset after executing the aforementioned cleaning and encoding process was subjected to the feature generation process explained in the next section.

**5.3 Feature Generation**

The standard feature columns in the dataset had data related to amounts and account balances in different phases of transactions at origin and destination accounts. The next step of data wrangling involves generation of processed columns as they would provide a better fit for the anomaly detection models. Thus, three additional columns were generated by applying a difference operation between the balance columns at origin and destination. The additional columns can be described as follows:

|  |  |
| --- | --- |
| **Feature** | **Description** |
| **diff\_newbalanceDest\_oldbalanceDest** | Difference between newbalanceDest & oldbalanceDest |
| **diff\_newbalanceDest\_newbalanceOrig** | Difference between newbalanceDest & newbalanceOrig |
| **diff\_newbalanceOrig\_oldbalanceOrg** | Difference between newbalanceOrig & oldbalanceOrg |

The dataset after applying the aforementioned steps can be viewed as follows:

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**5.4 Data Splitting & Down Sampling**

The dataset after undergoing the cleaning and transformation process had 4211109 transaction rows and 10 columns. The observed target rate with respect to “isFraud” column was just 0.19%. This is quite common in case of anomaly detection datasets in which a whole majority of transactions are legitimate in nature and the fraudulent ones lie hidden within them. The low target rate can be increased by down sampling the number of negative targets. However, it is highly advisable to split the dataset in training and validation sets first. The validation dataset should not be subjected to down sampling as the process of down sampling alters the reality of dataset. However, the

training set can be down sampled to swell the target rate to the required percentage. Moreover, the models trained over the down sampled training set must be validated against the unsampled validation set in order to make sure that the model can perform well in a real-time environment in which the overall number of fraudulent transactions will be very less as compared to legitimate ones. Thus, the original dataset was subjected to a stratified split in an 80-20 fashion with 80% of rows and positive targets in training set and rest of them in the validation set. The training set was down sampled by dropping the number of negative targets to 125,000. This increased the target rate to 5% within the down sampled training set.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Dataset** | **Total Number of rows** | **Number of 0s in target** | **Number of 1s in target** | **Target**  **Rate** |
| Training Set – Unsampled | 3368888 | 3362330 | 6558 | 0.19% |
| Training Set – Down Sampled | 131558 | 125000 | 6558 | 5% |
| Validation Set | 842221 | 840582 | 1639 | 0.19% |

**6. Feature Selection**

**6.1 Low Variance Check**

The first step to ensuring that the processed features will be a decent fit for the incoming modelling algorithms is ensuring that they have a considerable variance among their values. An acceptable variance value ensures that the data columns are covering a lot of cases and won’t overfit the model by limiting its view to a narrow and closed range of data. Thus, the down sampled training dataset was inputted to the low variance filter [20] in order to ensure that they have a decent spread of data. It was observed that all data columns had passed this check. This means that PaySim had generated the data with a wide ranged spread and the distribution was kept random in nature.

**6.2 Correlation Check**

Correlation is a statistical measure which quantifies how one column’s values are associated with increase or decrease in other columns’ values. Correlation is always measured in closed intervals between -1 and 1. High correlation can make the models consider two columns as duplicate and may learn the features in a one-dimensional manner which can affect the overall performance fatally. Thus, the features of the dataset were subjected to a correlation check among themselves and a matrix storing the correlation values was generated.

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The threshold of 0.85 for correlation was selected and one of the columns from pairs having correlation higher than 0.85 was removed. This resulted into removal of two columns: a) oldbalanceOrg and b) oldbalanceDest. Thus, the dataset was reduced to 6 feature columns after the correlation check.

**6.3 Feature Selection Using BORUTA Algorithm**

Boruta is a feature selection method that identifies all relevant features in a dataset by iteratively comparing the importance of actual features with randomized, noise-induced "shadow" features [15]. It works by training a machine learning model, typically a random forest, and assessing feature importance scores. Features with importance scores significantly higher than the best-performing shadow feature are deemed important, while those consistently lower are deemed unimportant and discarded [15]. The process is repeated until the algorithm converges or reaches a predefined number of iterations [15]. The features were ingested to a Boruta check with 100 iterations. The implementation was done using the BorutaPy library. It was observed that all 6 features had passed the Boruta check and all of them were processed towards the next phase – modelling.

**7. Modelling**

A bunch of machine learning algorithms were studied along with their impact during the literature survey [1][4][6][7]. However, the focus of modelling was narrowed down to 4 anomaly detection algorithms. The models were trained based on the down sampled training dataset with the features which had passed all 3 aforementioned checks. The algorithms were executed and the trained models were tested against the unsampled validation dataset in order to ensure that they perform well in real-time situations.

**7.1 Isolation Forest**

Isolation Forest is an anomaly detection algorithm that identifies outliers by isolating data points [16]. It works by randomly selecting features and then randomly choosing split values between the minimum and maximum values of the selected features [16]. This process recursively partitions the data, creating a tree structure where anomalies are more likely to be isolated early due to their sparse nature. The path

length from the root to the point of isolation is shorter for anomalies than for normal points [16]. The algorithm calculates an anomaly score based on the average path length across many trees. Points with shorter average path lengths are considered more anomalous. Isolation Forest is efficient and works well with high-dimensional data [16].

**7.2 One Class Support Vector Machine**

One-Class SVM (Support Vector Machine) is an unsupervised learning algorithm used for anomaly detection [17]. It models the "normal" data by finding a decision boundary that best encompasses the majority of the training data points in a high-dimensional space [17]. Points that fall outside this boundary are classified as anomalies. The algorithm works by mapping the input data into a higher-dimensional feature space using a kernel function, where it seeks to maximize the margin around the data cluster [17]. It is particularly effective when the training data contains mostly normal examples and few or no anomalies. One-Class SVM is widely used in scenarios like fraud detection and outlier detection [17].

**7.3 Auto Encoder Neural Networks**

An autoencoder is a type of neural network used for unsupervised learning, primarily for tasks like dimensionality reduction, feature learning, and anomaly detection [18]. It consists of two main parts: an encoder that compresses the input data into a lower-dimensional latent space, and a decoder that reconstructs the original data from this compressed representation [18]. The network is trained to minimize the reconstruction error, meaning the difference between the input and the output. In anomaly detection, autoencoders learn to reconstruct normal data well but struggle with anomalies, resulting in higher reconstruction errors [18]. The architecture can be adapted with additional layers or different activation functions to suit various data types, including images and time series [18]. Autoencoders are powerful tools for capturing complex patterns and structures within data, making them useful in various machine learning applications.

**7.4 Elliptic Envelope**

Elliptic Envelope is an anomaly detection algorithm that assumes the data follows a Gaussian distribution and models it using a multivariate normal distribution. It fits an ellipsoid to the central data points, capturing the underlying data distribution. Data points that lie outside this ellipsoid are considered anomalies. The algorithm estimates the mean and covariance of the data and uses them to define the ellipsoid’s shape and orientation. Elliptic Envelope is effective when the data is approximately Gaussian but may struggle with complex or non-Gaussian distributions. It is particularly useful for detecting outliers in multivariate data, such as financial or environmental datasets.

**7.5 Performance Metrics**

The following performance metrics were generated from testing the performance of the aforementioned models on the unsampled validation data:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Model** | **Accuracy** | **Precision** | **Recall** | **AUC** |
| Isolation Forest | 95.77% | 1.2% | 26.36% | 61.13% |
| One Class SVM | 95.38% | 0.7% | 17.88% | 56.70% |
| AutoEncoders | 99.36% | 0.19% | 100% | 50% |
| **Elliptic Envelope** | **92.47%** | **59.25%** | **87.73%** | **81.29%** |

The aforementioned statistics confirm that Elliptic Envelope Algorithm performs the best when it comes to accuracy, precision, recall and AUC. The isolation forest fails to identify the positive samples as the precision is very low. Autoencoders have clearly under fitted as all transactions in validation set were classified as legitimate. The One class SVM fails to identify a lot of fraudulent transactions. The Gaussian properties of Elliptic Envelope seem to fit the fraudulent transactions.

**8. Conclusion**

The analysis and modelling have proven that the dataset provided by [12][14] can be processed using an elliptic envelope model for analyzing a high volume of fraudulent transactions. However, we believe that purely relying on a machine learning based approach may cause the accuracy to degrade gradually due to data drifts. Thus, constant monitoring of the performance metrics of model is required and the model needs to be refined over new datasets in case of drifts. Moreover, we can increase the confidence of determining fraudulent transactions based on certain observations from the exploratory data analysis phase. An example can be the sequence of fraudulent transactions. Every fraudulent transaction has occurred with a consecutive dependency on a previous transaction. The “Type” column in case of fraudulent transactions carries only two unique values: TRANSFER and CASH OUT. Thus, if the check of fraudulent transactions using machine learning can be just limited to the sequence of transactions of these two types. These conditions can be manually added apart from the results in order to be perfectly sure if an incoming transaction is fraudulent or not.

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